

## Gravitation and Kepler's Laws

- Newton's law of universal gravitation states that each pair of objects in the universe exerts a gravitational force of attraction on each other that is directly proportional to each mass and inversely proportional to the distance between their centers squared. The constant of proportionality is  $G$ .

$$F_g = G \frac{m_1 m_2}{R^2}$$

- It can be seen that by substituting the mass of a large object (like the earth) and the mass of the small object (like a stone) resting on it for the two masses and substituting the radius of the large object for the distance, the acceleration due to gravity on the surface can be calculated for any planet, moon, asteroid, comet, etc.

$$F_g = mg = m \left( G \frac{m_o}{R^2} \right) = mg_o$$

where  $m_o$  is the mass of the large object,  $m$  is the mass of the small object and  $R$  is the radius of the large object.

- Johannes Kepler observed three things about the motion of our planets around the sun:
  - The planets revolve around the sun in elliptical orbits with the sun at one of the foci.
  - An imaginary line drawn from the sun to a planet will sweep out equal areas in equal times.
  - The ratio of a each planet's mean orbital radius cubed to the period of the orbit squared is a

constant.  $\frac{R^3}{T^2} = \text{a constant}$

- Isaac Newton realized that it was gravity that provides the centripetal for orbital motion and verified Kepler's observed relation by setting the expression for gravitational force equal to the expression for centripetal force:

$$G \frac{m_s m_p}{R^2} = \frac{m_p v^2}{R}$$

- The above equation can be simplified to yield:

$$\frac{G}{4\pi^2} m_o = \frac{R^3}{T^2} \text{ where } m_o \text{ is the mass of the object being orbited.}$$

- Orbital characteristics are determined by the mass of the object being orbited but not by the mass of the object that is orbiting.
- The mass of an object that is in orbit is **NOT UNIMPORTANT** since it determines the amount of energy required to obtain orbit.
- Henry Cavendish determined the value of the gravitational constant ( $G$ ) by careful measurement of the gravitational force between heavy lead spheres.